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RUN-UP FLOW OF A CONDUCTING VISCO-ELASTIC [OLDROYD (1958) MODEL] LIQUID IN A LONG UNIFORM CIRCULAR TUBE

Anil Tripathi, Ph. D.
Department of Mathematics, K.K. (P.G.) College, Etawah (U.P.), India.
Email ID: atkkdc@gmail.com

Abstract

In the present paper Laplace Transform Technique has been applied to analyse the run-up flow of a conducting visco-elastic [Oldroyd (1958) type] liquid through an infinitely long uniform right circular cylinder under the influence of uniform magnetic field applied perpendicularly to the flow of the liquid. The motion is initially generated by constant pressure gradient along the axis of circular tube. When the flow is fully developed, the pressure gradient is suddenly withdrawn whereas the wall of the cylinder is impulsively started simultaneously. Expressions for velocity field, flow rate and the shear stress on the wall have been obtained. The result for Kuvshiniski visco-elastic, Rivlin-Ericksen visco-elastic and ordinary viscous fluid flows are also deduced as the limiting cases. If the magnetic field is with-drawn, all corresponding results for flow of viscous and various types of visco-elastic fluids can be determined.

INTRODUCTION

In many configurations, in technology and in nature we continually encounter run-up flow of fluids between rigid boundaries. Due to their importance many researchers have investigated such flows for viscous and visco-elastic fluids such as Kazakia and Rivlin[1], Rivlin[3], Ramacharyulu and Raja[4], Singh and Srivastava[5], Singh[6], studied the run-up flow of Oldroyd[2] model visco-elastic liquid through porous medium in along the right circular cylinder, Nayak, Dash and Panda[7] studied MHD flow of a visco-elastic fluid along vertical porous surface with comical reaction, Chaudry, Dhar and Dey[8] have discussed visco-elastic MHD flow through a porous medium bounded by horizontal parallel plates moving in opposite direction in presence of heat and mass transfer,Tripathi[9] studied unsteady MHD flow of a conducting visco-elastic liquid through porous medium between two finite co-axial right circular cylinders.

The object of this paper is to study the run-up flow of Oldroyd visco-elastic liquid through an infinitely long right circular cylinder with the application of uniform magnetic field applied perpendicularly to the direction of fluid with the help of Laplace transform.

Expressions for velocity field, flow rate and the shear stress on the circular wall are obtained. The results for Kuvshiniski, Rivlin-Ericksen visco-elastic liquids and ordinary viscous fluid flows are also deduced as the limiting cases. If magnetic field is withdrawn, all corresponding results for purely viscous fluid and various type of visco-elastic liquids can be determined.

FORMULATION OF THE PROBLEM

We consider the flow of an incompressible elastico-viscous [Oldroyd (1958) model] liquid for which the strain relation is given by

$$P_{ik} = -P\delta_{ik} + P_k^{i'}$$

Consider the flow of a visco-elastic liquid in a circular cylinder of radious a. Referring the problem to cylindrical polar coordinates (r, θ, z) we take the z-axis along the axis of the cylinder. It is assumed that $W_r = W_\theta = 0$ and $W_z = W(r, t)$.

With the above assumptions the equation of motion relevant to the problem is:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t} = -\frac{1}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \nu\left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right) - \frac{\sigma B_0^2}{\rho}\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)W \quad \ldots (1)$$

where ρ is the density, ν the kinematic viscosity, σ the conductivity of fluid B_0 is the electromagnetic induction.

At t=0, a constant pressure gradient is impressed on the system, when the flow is fully developed, the pressure gradient is suddenly withdrawn and at the same time the wall starts moving with a constant velocity W_0 parallel to itself.

Introducing the following non-dimensional quantities:

$$W^* = \frac{a}{\nu}W, \quad t^* = \frac{\nu}{a^2}t, \quad P^* = \frac{a^2}{\rho\nu^2}P, \quad z^* = \frac{z}{a},$$

$$r^* = \frac{r}{a}, \quad \lambda_1^* = \frac{\nu}{a^2}\lambda_1, \quad \mu_1^* = \frac{\nu}{a^2}\mu_1,$$

in equation (1) we get after dropping the stars:

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t} = -\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} + \left(1 + \mu_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right)$$

$$-M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)W \qquad \ldots(2)$$

where $M = aB_0\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number)

INITIAL STATE FOR FLOW

The steady state flow when $-\frac{\partial p}{\partial z} = C$ (constant) is given by the momentum equation

$$0 = C + \left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right) - M^2 W \qquad \ldots(3)$$

together with the boundary conditions

$$\left.\begin{aligned} W &= 0, && \text{at } r = 1 \\ W &= \text{finite}, && \text{at } r = 0 \end{aligned}\right\} \qquad \ldots(4)$$

The solution of (3) with the help of (4) is

$$W = CK\left[1 - \frac{I_0(Mr)}{I_0(M)}\right] = f(r) \ \ (\text{say}) \qquad \ldots(5)$$

where $I_0(r)$ is the modified Bessel's function of order zero.

RUN-UP FLOW

The constant pressure gradient is suddenly withdrawn and at the same time the wall starts moving with a constant velocity W_0.

Now the velocity field satisfied the equation

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\frac{\partial W}{\partial t} = \left(1+\mu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 W}{\partial r^2} + \frac{1}{r}\frac{\partial W}{\partial r}\right) - M^2\left(1+\lambda_1\frac{\partial}{\partial t}\right)W \qquad \ldots(6)$$

The initial and boundary conditions are:

$$t = 0 \quad , \quad W = f(r) \qquad \ldots(7)$$

$$\left.\begin{aligned} & W = W_0, && \text{at } r = 1 \\ t > 0 \quad , & \\ & W = \text{finite}, && \text{at } r = 0 \end{aligned}\right\} \qquad \ldots(8)$$

Applying Laplace transform to equation (6) we get,

$$\frac{d^2\bar{W}}{dr^2}+\frac{1}{r}\frac{d\bar{W}}{dr}-\lambda^2\left\{\bar{W}-\frac{C[1+\lambda_1(M^2+1)]}{M^2(1+\lambda_1 s)(s+M^2)}\right\}$$

$$=M^2\frac{(1+\lambda_1 s)(\lambda_1-\mu_1)}{(1+\mu_1 s)}\times\frac{I_0(Mr)}{I_0(M)} \qquad \ldots(9)$$

$$\text{where} \qquad \lambda^2=\frac{(1+\lambda_1 s)(s+M^2)}{(1+\mu_1 s)}$$

Together with the reduced boundary conditions:

$$\left.\begin{aligned} &\bar{W}=\frac{W_0}{s}, \quad \text{at} \quad r=1 \\ &\text{for} \quad t>0 \quad , \\ &\bar{W}=\text{finite}, \quad \text{at} \quad r=0 \\ &\text{where} \quad \bar{W}(s,t)=\int_0^\infty W(r,t)e^{-st}dt \end{aligned}\right\} \qquad \ldots(10)$$

Solution of equation (9) subject to conditions (10) is :

$$\bar{W}=\left[\left\{\frac{W_0}{s}+\frac{C}{sM^2}-\frac{C}{M^2}\left\{\frac{1+\lambda_1 s+\lambda_1 M^2}{(1+\lambda_1 s)(s+M^2)}\right\}\right\}\right]\frac{I_0(\lambda r)}{I_0(\lambda)}$$

$$-\frac{CK}{s}\frac{I_0(Mr)}{I_0(M)}+M^2\left[\frac{1+\lambda_1 s+\lambda_1 M^2}{(1+\lambda_1 s)(s+M^2)}\right] \qquad \ldots(11)$$

The inverse Laplace transform corresponding to equation (11), we get the velocity of visco-elastic liquid is

$$W(r,t)=W_0\frac{I_0(Mr)}{I_0(M)}+2\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{\alpha_n\left(1+\mu_1 S_n^i\right)^2 e^{S_n^i t}}{\left[1+M^2(\lambda_1+\mu_1)+2\lambda_1 S_n^i+\lambda_1\mu_1 {S_n^i}^2\right]}$$

$$\times\left[\frac{W_0}{S_n^i}+\frac{C}{M^2 S_n^i}-\frac{C}{M^2}\frac{\left(1+\lambda_1 S_n^i+\lambda_1 M^2\right)}{(1+\lambda_1 S_n^i)(M^2+S_n^i)}\right]\frac{J_0(\alpha_n r)}{J_1(\alpha_n)} \qquad \ldots(12)$$

Where S_n^i are the roots of the quadratic equation

$$\frac{\left(1+\lambda_1 S_n^i\right)\left(M^2+S_n^i\right)}{1+\mu_1 S_n^i}=-\alpha_n^2 \qquad \ldots(13)$$

α_n are the successive roots of the equation $J_0(\alpha)=0$ and J_0, J_1 are the Bessel's function of order zero and one respectively. The flow rate is given by

$$Q = 2\pi\left[\frac{W_0}{M}\frac{I_1(Mr)}{I_0(M)} + 2\sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\frac{\left(1+\mu_1 S_n^i\right)^2 e^{S_n^i t}}{[1+M^2(\lambda_1-\mu_1)+2\lambda_1+S_n^i+\lambda_1\mu_1 S_n^i]^2}\right.$$
$$\left.\times\left\{\frac{W_0}{S_n^i}+\frac{C}{M^2 S_n^i}-\frac{C}{K}\frac{\left(1+\lambda_1 S_n^i+\lambda_1 M^2\right)}{(1+\lambda_1 S_n^i)(M^2+S_n^i)}\right\}\right] \quad \ldots(14)$$

The shear stress on the wall is given by

$$\tau = \mu M W_0\frac{I_1(Mr)}{I_0(M)} - 2\sum_{n=1}^{\infty}\sum_{i=1}^{\infty}\frac{\alpha_n^2\left(1+\mu_1 S_n^i\right)^2\left[1-(\lambda_1-\mu_1)S_n^i\right]e^{S_n^i t}}{\left[1+M^2(\lambda_1-\mu_1)+2\lambda_1 S_n^i+\lambda_1\mu_1 {S_n^i}^2\right]}$$
$$\times\left[\frac{W_0}{S_n^i}+\frac{C}{M^2 S_n^i}-\frac{C}{M^2}\frac{\left(1+\lambda_1 S_n^i+\lambda_1 M^2\right)}{(1+\lambda_1 S_n^i)(M^2+S_n^i)}\right] \quad \ldots(15)$$

This is the flow with the boundary wall moving the velocity $W_0$and in the absence of pressure gradient.

STEADY CASE

When $t \to \infty$ from equations (12), (14) and (15), we have

$$W(r,\infty) = W_0\frac{I_0(Mr)}{I_0(M)} \quad \ldots(16)$$

$$Q = 2\pi W_0 M\frac{I_1(Mr)}{I_0(M)} \quad \ldots(17)$$

$$\tau = \mu M W_0\frac{I_1(Mr)}{I_0(M)} \quad \ldots(18)$$

PARTICULAR CASE

Case-I : If $\mu_1 \to 0,$ all corresponding result for Kuvshiniski type visco-elastic Liquid can be determined.

Case-II : If $\lambda_1 \to 0,$ all corresponding result for Rivlin-Ericksen type visco-elastic Liquid can be obtained.

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